Student Number

Sydney Girls High School
2024
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION
Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

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Total Marks:

100

Section I – 10 marks (pages 3–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–34)

- Attempt Questions 11–36
- Allow about 2 hours and 45 minutes for this section

THIS IS A TRIAL PAPER ONLY

It does not necessarily reflect the format or the content of the 2020 HSC Examination Paper in this subject.

Question	M.C	11-20	21-24	25-29	30-34	35-37	Total
Marks	/10	/31	/14	/16	/14	/15	/100

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

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1 What is $\int x^2(2x^3+1)^5\,dx$.

A. $\frac{x^3}{18}\left(2x^3+1\right)^6+C$

B. $\frac{1}{6}\left(2x^3+1\right)^6+C$

C. $\frac{x^2}{36}\left(2x^3+1\right)^6+C$

D. $\frac{1}{36}\left(2x^3+1\right)^6+C$

2 What is the domain of the function $\ln(x+2)+\sqrt{9-x}$?

A. $(-2,9)$

B. $[-2,9]$

C. $(-2,9]$

D. $[-2,9)$

3 Find $\int (e+\sin 2x)\ dx$.

A. $\frac{e^2}{2} - \frac{1}{2}\cos 2x + c$

B. $ex - \frac{1}{2}\cos 2x + c$

C. $\frac{e^2}{2} - 2\cos 2x + c$

D. $ex - 2\cos 2x + c$

4 Which of the following is an expression for $\frac{d}{dx}\log_7 4x$?

A. $\frac{7}{4x\ln 7}$

B. $\frac{1}{x\ln 7}$

C. $\frac{4}{x\ln 7}$

D. $\frac{\ln 7}{\ln 4x}$

5 Consider the series $\log_a 54 + \log_a 18 + \log_a 6 + \ldots\ldots\ldots\ldots.$

Which of the following statements best describe the series?

A. A geometric series with common ratio of -3.

B. A geometric series with common ratio of $-\log_a 3$.

C. An arithmetic series with common difference -3

D. An arithmetic series with common difference of $-\log_a 3$.

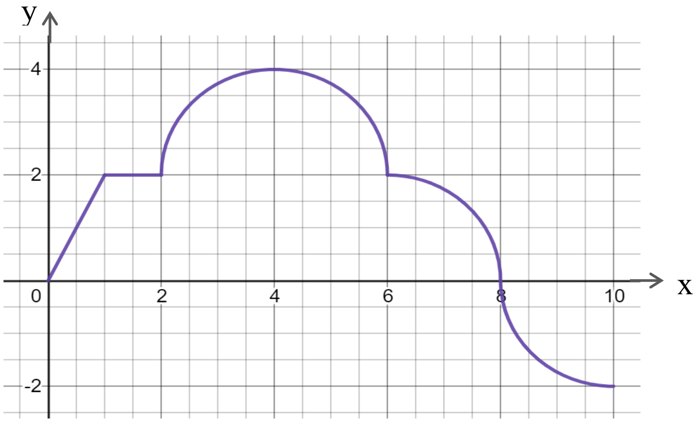
6 Bella recently did a Maths test and an English test. The class scores on each test were normally distributed. The mean on the English test was 60%, and the standard deviation was 10%. Bella's mark was 72%.

The mean on the Maths test was 73%, and the standard deviation was 5%.

What was Bella's mark in the Maths test, if her *z*-scores in English and Maths were equal?

A. 72%
B. 78%
C. 79%
D. 83%

7 The diagram shows the graph of $y = f(x)$, which is made of two-line segments, a semi-circle and two quarters of a circle.



Which of the following is the value of $\int_0^{10} f(x)dx$?

A. $2\pi + 11$

B. $4\pi + 11$

C. $4\pi + 12$

D. $2\pi + 12$

8 Let $h(x)=f(g(x))$, with $f(4)=1$, $f'(4)=6$,$g(2)=4$ and $g'(2)=4$. What is the equation of the tangent to the graph of $y=h(x)$ at $x=2$?

A. $y=x-6$

B. $y=24x-2$

C. $y=6x-11$

D. $y=24x-47$

9 Simplify $\cot\left(270^\circ+x^\circ\right)\cos\left(-x^\circ\right)+\cos\left(90^\circ+x^\circ\right)$

A. $-2\sin x$

B. $\sin x+\cos x$

C. 0

D. $2\sin x$

10 A biased four-sided die is rolled.

The following table gives the probability of each score.

Score	1	2	3	4
Probability	$\frac{7}{16}$	$\frac{5}{16}$	3^k	3^{2k+1}

What is the probability of rolling 3?

A. $\frac{1}{4}$

B. $\frac{1}{6}$

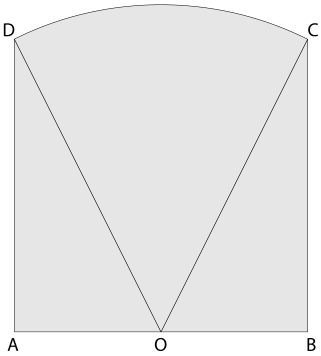
C. $\frac{1}{8}$

D. $\frac{1}{5}$

Question 11 (5 marks)

A shape consists of three sides of a square together with a sector, ODC, where O is the midpoint of AB, as shown below (not to scale).

$$DA = AB = BC = 2 \text{ cm}$$



a) Find the exact length of DO. **(1)**

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b) If $\angle DOC = \theta$, show that $\cos\theta = \dfrac{3}{5}$. **(2)**

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c) Hence find the perimeter of the shape $ABCD$ correct to 2 decimal places. **(2)**

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Question 12 (3 marks)

The following table shows the probability distribution of a discrete random variable with an expected value of two.

x	0	1	2	3	4
$P(X=x)$	0.2	0.1	a	b	0.2

a) Calculate the values of a and b. **(2)**

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b) Hence calculate the standard deviation, correct to two decimal places. **(1)**

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Question 13 (2 marks)

Show that the curve $y = 3x^2 - 5\ln x$ is concave up for all values of $x > 0$. **(2)**

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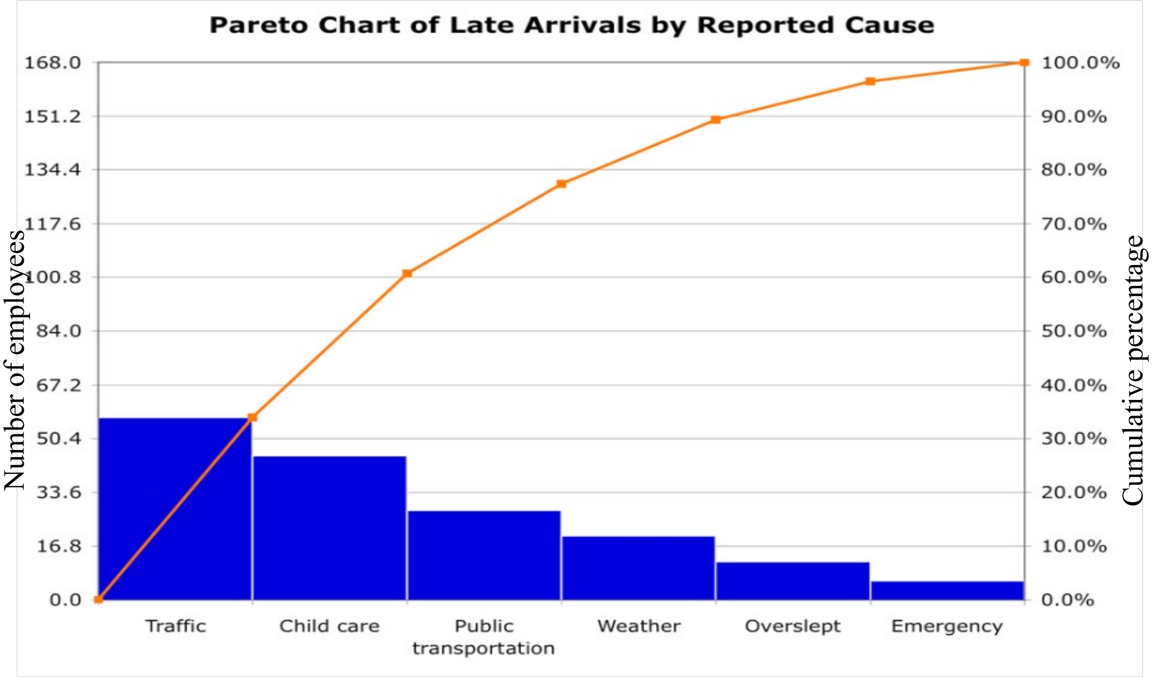
Question 14 (1 marks)

A company owner collected data related to the reasons given by employees for being late to work.

The Pareto chart shows the data collected.



Estimate the percentage of employees that were late because of public transport. **(1)**

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Question 15 (2 marks)

Differentiate $e^{4x}\ln(x^2)$. **(2)**

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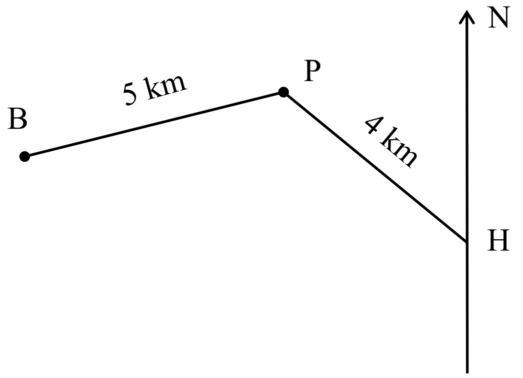
Question 16 (5 marks)

Kate starts riding her bike from her house H. She rides 4 km on a bearing 320° to the park P then rides 5 km on a bearing 250° to the beach B.



i) Show that the angle HPB is 110°. **(1)**

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b) Find the distance BH correct to three decimal places. **(2)**

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c) Find the bearing of Kate's house H from the beach B. **(2)**

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Question 17 (3 marks)

If $0 \leq \theta \leq 2\pi$, solve $\sqrt{2}\cos\left(2\theta - \frac{\pi}{3}\right) - 1 = 0$. **(3)**

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Question 18 (2 marks)

Explain why the geometric series shown below does not have a limiting sum. **(2)**

$$3 + \frac{3}{\sqrt{3}-1} + \frac{3}{\left(\sqrt{3}-1\right)^2} + ...$$

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Question 19 (2 marks)

The future value of an annuity when $1 is invested at the start of each period is shown in the table below.

Future value of $1						
Number of periods	Interest rate per period					
	1%	2%	3%	4%	5%	6%
1	$1.0100	$1.0200	$1.0300	$1.0400	$1.0500	$1.0600
2	$2.0301	$2.0604	$2.0909	$2.1216	$2.1525	$2.1836
3	$3.0604	$3.1216	$3.1836	$3.2465	$3.3101	$3.3746
4	$4.1010	$4.2040	$4.3091	$4.4163	$4.5256	$4.6371
5	$5.1520	$5.3081	$5.4684	$5.6330	$5.8019	$5.9753
6	$6.2135	$6.4343	$6.6625	$6.8983	$7.1420	$7.3938

Jamie deposits $800 into a savings account at the start of each month for 6 months.
After the 6th deposit, Jamie stops making deposits but leaves the money in the savings account until exactly 12 months from the first deposit.
The interest rate on her savings account is 12% per annum, compounded monthly.
What is the balance of Jamie's savings account at the end of the 12 months? **(2)**

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Question 20 (6 marks)

Given a function:

$$f(x) = \begin{cases} \dfrac{3x}{4}(2-x) \text{ for } 0 \le x \le 2 \\ 0 \text{ otherwise} \end{cases}$$

i) Prove that $f(x)$ represents a probability density function. **(2)**

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ii) State the mode of the distribution. **(1)**

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iii) Explain why the median is equal to the mode in this distribution. **(1)**

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iv) Find $P(X > 1.5)$. **(2)**

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Mathematics Advanced

Student Number

Section II

Answer Booklet 2

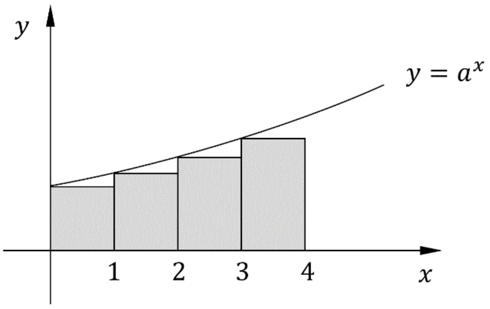
Booklet 2 - Attempt Questions 21 – 37

Instructions Answer the questions in the spaces provided.

- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the question paper.
  If you use this space, clearly indicate which question you are answering.

Question 21 (4 marks)

The diagram shows the graph of $y = a^x$, where $a > 1$. Also shown on the diagram are 4 inner rectangles of width 1 unit.



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i) Show that the area of the 4 rectangles is $\dfrac{a^4 - 1}{a - 1}\ u^2$. **(2)**

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ii) By finding the area under the curve $y = a^x$ between $x = 0$ and $x = 4$, and using the result in part (i), prove that $\ln a < a - 1$. **(2)**

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Question 22 (4 marks)

A continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} 3^{-x} \ln 3 & \text{for } x \geq 0 \\ 0 & \text{for all other values of } x \end{cases}$$

i) Find the cumulative distribution function $F(x)$. **(2)**

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ii) Find the exact value of the third quartile. **(2)**

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Question 23 (2 marks)

Evaluate $\int_1^6 |2x-6| \, dx$. **(2)**

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Question 24 (4 marks)

Sketch the graph of the curve $y = \frac{5x^2 e^{\frac{x}{2}}}{3}$ showing the coordinates of the turning points and their nature. **(4)**

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Question 25 (2 marks)

Differentiate $\ln\left(3\sin\frac{x}{2}\right)^3$. **(2)**

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Question 26 (2 marks)

If $\int_0^3 g(x)\ dx = 11$, find $\int_0^3 \left(\frac{1}{2}g(x) + 3x\right) dx$. **(2)**

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Question 27 (3 marks)

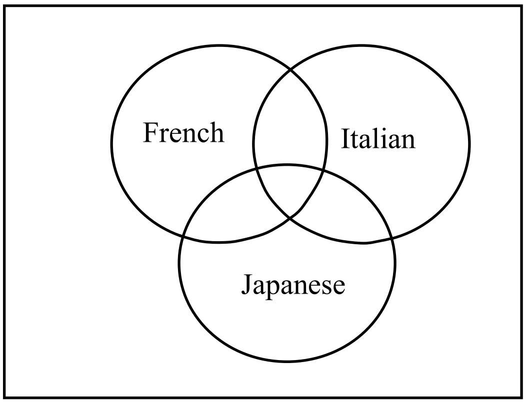
Last Sunday, 45 people were surveyed about three types of restaurants that they prefer to attend on their next birthday.

Of them, 26 chose French, 22 chose Italian, 18 Japanese.

It is also found that 11 chose French and Italian, 10 chose French and Japanese and 7 chose Italian and Japanese.

Only 4 people chose all three types of restaurants.

a) Represent this information in the Venn Diagram below. **(1)**



i) A person from those who were surveyed last Sunday is to be selected at random. What is the probability that this person chose French or Italian but not Japanese? **(1)**

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ii) Two people are chosen at random, what is the probability of both chose French only? **(1)**

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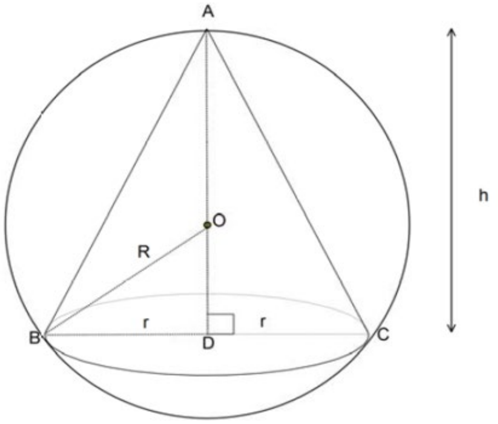
Question 28 (3 marks)

Prove that $\left(\cot\theta + \operatorname{cosec}\theta\right)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. **(3)**

Question 29 (6 marks)

Consider the following diagram with the cone inscribed in a sphere.



i) If r is the radius of the cone show that $r^2 = 2hR - h^2$. **(1)**

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ii) Show that the volume of the cone that can be inscribed in a sphere of radius R is given by $V = \frac{1}{3}\pi\left(2h^2R - h^3\right)$ where h is the height of the inscribed cone. **(1)**

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__Question 29 continued on the next page__

<u>Question 29 continued</u>

iii) Show that the volume of the largest cone is $\frac{8}{27}$ of the volume of the sphere. **(4)**

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Question 30 (2 marks) (The table on page 27 can be used for this question)

Madelyn is a long jumper training for the national age championships. To be able to compete at the national age championships her longest jump must be able to be beaten by less than 0.17% of the general population.
The distance jumped by the general population is normally distributed with a mean of 3.48 metres and standard deviation of 1.08 metres.
What is the minimum distance that Madelyn needs to jump, correct to the nearest centimetre?

Table : The standard normal distribution

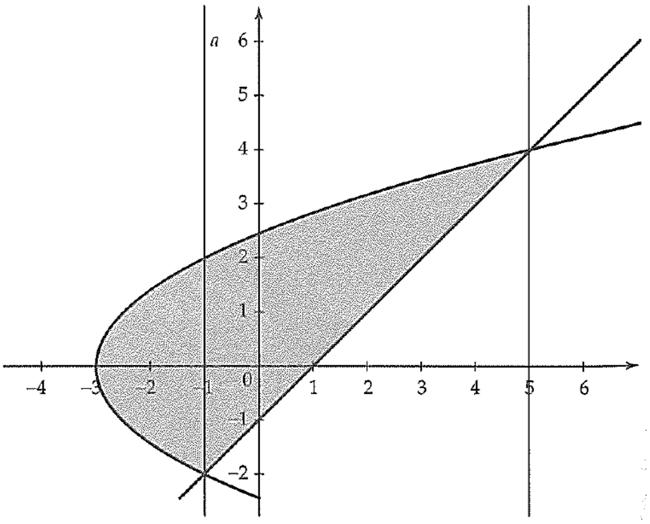
The table below provides some values of the probabilities for the standard normal distribution.

$$\text{i.e.}\quad \Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \phi(t)dt$$

z	**+0.00**	**+0.01**	**+0.02**	**+0.03**	**+0.04**	**+0.05**	**+0.06**	**+0.07**	**+0.08**	**+0.09**
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56360	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

Question 31 (4 marks)

Determine the area of the region enclosed by $y^2 = 2x + 6$ and $y = x - 1$.



Question 32 (4 marks)

Madison is learning to drive. Her first lesson is 10 minutes long. Her second lesson is 15 minutes long. Each subsequent lesson is 5 minutes longer than the previous lesson.

i) How long will Madison's fifteenth lesson be? **(1)**

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ii) How many hours of lessons will Madison have completed after her fifteenth lesson? **(1)**

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iii) During which lesson will Madison have completed a total of 40 hours of driving lessons? **(2)**

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Question 33 (2 marks)

Solve the equation $\log_2\left(2-2x\right)=\log_{\sqrt{2}}\left(1-x\right)$. **(2)**

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Question 34 (2 marks)

Differentiate $y=\left(\sin^3 2x^{\circ}\right)$. **(2)**

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Question 35 (7 marks)

Marge borrows \$50 000 in order to buy a car. The loan attracts an interest of just 0.5% per month. The company also offers an interest free period for the first six months. However, the first payment is due at the end of the first month. Marge agrees to repay the loan over 10 years, by making 120 equal monthly repayments of $\$M$. Let A_n be the amount owing at the end of nth repayment, then

i) Show that $A_6 = 50\ 000 - 6M$ **(1)**

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ii) Show that $A_8 = (50\ 000 - 6M) \times 1.005^2 - M(1.005 + 1)$ **(2)**

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iii) Hence derive the expression for A_{120} in simplest form. **(2)**

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iv) Find the value of the monthly repayment to the nearest cent. **(2)**

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Question 36 (5 marks)

A particle is moving in a straight line. It's displacement, x metres, from the origin, *O* at time *t* seconds, where $t \geq 0$, is given by $x = 4 + 3te^{-2t}$

i) Show the velocity of the particle is given by $\frac{dx}{dt} = 3e^{-2t}\left(1 - 2t\right)$. **(2)**

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ii) Show that the particle is at rest when $t = \frac{1}{2}$. **(1)**

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iii) Find the greatest possible total distance the particle could travel. **(2)**

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Question 37 (3 marks)

Sally contributes $7200 each year into a superannuation fund for the first 15 years of her working life. For the next 20 years until retirement, she decides to increase this, and invest a total of $10000 each year. Each contribution is paid at the beginning of the year.

If the investment earns 7% per annum paid yearly over whole period, how much will her investment be upon retiring? (Show all your working) **(3)**

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End of the paper

Multiple Choice Answer Sheet

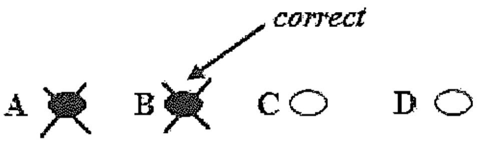
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:



Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ○ D ●

2. A ○ B ○ C ● D ○

3. A ○ B ● C ○ D ○

4. A ○ B ● C ○ D ○

5. A ○ B ○ C ○ D ●

6. A ○ B ○ C ● D ○

7. A ● B ○ C ○ D ○

8. A ○ B ○ C ○ D ●

9. A ● B ○ C ○ D ○

10. A ○ B ● C ○ D ○

2024 Advanced Trial MC

1. $\frac{1}{6}\int 6x^2(2x^3+1)^5\,dx$

$$= \frac{1}{6}\,\frac{(2x^3+1)^6}{6} + c$$

$$= \frac{(2x^3+1)^6}{36} + c$$ Ⓓ

2. $x > -2$

$9 - x \geqslant 0$

$-x \geqslant -9$

$x \leqslant 9$

$D:\ x \in (-2, 9]$ Ⓒ

3) $\int e + \sin 2x \; dx$

$$= ex - \frac{\cos 2x}{2} + c$$ Ⓑ

4) $\log_7 4x = \frac{\ln 4x}{\ln 7}$

$$\frac{d}{dx}\frac{\ln 4x}{\ln 7} = \frac{1}{\ln 7} \times \frac{\cancel{4}}{\cancel{4}x}$$

$$= \frac{1}{x \ln 7}$$ Ⓑ

5) $\log_a 18 - \log_a 54$

$= \log_a \frac{18}{54}$

$= \log_a \frac{1}{3}$

$= \log_a 3^{-1}$

$= -\log_a 3$ Ⓓ

6) $\bar{x}_E = 60\%$

$\sigma_E = 10\%$

$Z = \frac{12}{10} = 1.2$

$\bar{x}_M = 73\%$

$\sigma_M = 5\%$

$M = 79\%$ Ⓒ

7) $\int_0^{10} f(x)\,dx = \frac{1}{2} \times 2(2+1) + \frac{1}{2} \times \pi \times 4 + 4 \times 2$

$= 3 + 2\pi + 8$

$= 11 + 2\pi$ Ⓐ

8) $h'(x) = f'(g(x)) \cdot g'(x)$

$= 6 \times 4$

$= 24$

$y - y_1 = 24(x - x_1)$

$y_1 = f(4)$

$= 1$

$y - 1 = 24(x - 2)$

$y = 24x - 48 + 1$

$y = 24x - 47$ Ⓓ

9) $\cot(360 - 90 + x)$

$\cot(360 - (90 - x)$

$- \cot(90 - x)$

$= -\tan x$

$\cos(90 + x) = \cos(180 - 90 + x)$

$= \cos(180 - (90 - x))$

$= -\cos(90 - x)$

$= -\sin x$

$-\tan x \times \cos x - \sin x$

$= -\frac{\sin x}{\cos x} \times \cos x - \sin x$

$= -2\sin x$ Ⓐ

10) $\frac{7}{16} + \frac{5}{16} + 3^{k} + 3^{2k+1} = 1$

$\frac{12}{16} + 3^{k} + 3^{2k+1} - 1 = 0$

$3^{k} + 3^{2k+1} - \frac{1}{4} = 0$

$4(3^{k}) + 4(3^{2k+1}) - 1 = 0$

$4(3^{2k} \cdot 3) + 4(3^{k}) - 1 = 0$ Let $m = 3^{k}$

$12(3^{2k}) + 4(3^{k}) - 1 = 0$

$12m^{2} + 4m - 1 = 0$

$12m^{2} + 6m - 2m - 1 = 0$

$6m(2m+1) - (2m+1) = 0$

$(2m+1)(6m-1) = 0$

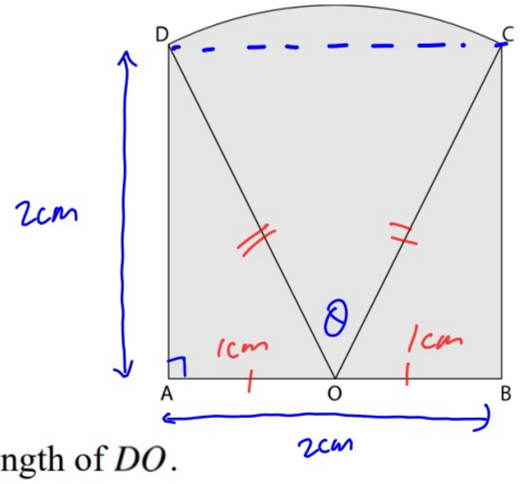
$m \neq -\frac{1}{2}$ $m = \frac{1}{6}$

$3^{k} = \frac{1}{6}$ Ⓑ

Question 11 (5 marks)

A shape consists of three sides of a square together with a sector, ODC, where O is the midpoint of AB, as shown below (not to scale).

$$DA = AB = BC = 2 \text{ cm}$$



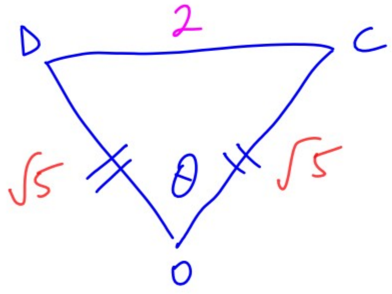
a) Find the exact length of DO.

$DO = \sqrt{2^2+1^2} = \sqrt{5}$

Mark	Criteria
1	Correct solution

Done well.

b) If $\angle DOC = \theta$, show that $\cos\theta = \dfrac{3}{5}$.



$$\cos\theta = \frac{(\sqrt{5})^2 + (\sqrt{5})^2 - 2^2}{2(\sqrt{5})(\sqrt{5})} \quad ①$$

$$\cos\theta = \frac{6}{10} = \frac{3}{5} \quad ①$$

Mark	Criteria
2	Correct solution
1	Uses the cosine rule but fails to obtain the correct solution. **OR** Uses the $\tan^{-1} 2$ but fails to arrive at an expression for $\cos\theta$.

Comments

Many students overcomplicated their solution by not using cos rule.

c) Hence find the perimeter of the shape $ABCD$ correct to 2 decimal places. **(2)**

$P = 2 \times 3 + r\theta$

$= 6 + \sqrt{5} \times \cos^{-1}\left(\frac{3}{5}\right) \quad ①$

$\doteqdot 8.07$ cm $\quad ①$

Mark	Criteria
2	Correct solution
1	Obtains the correct expression for the P but fails to arrive at the correct final answer.

Comments

Many students had their calculator in degrees mode and thus failed to get full marks

Question 12 (3 marks)

The following table shows the probability distribution of a discrete random variable with an expected value of two.

x	0	1	2	3	4
$P(X=x)$	0.2	0.1	a	b	0.2

a) Calculate the values of a and b.

Mark	Criteria
2	Correct solution.
1	Incorrectly obtains the values of a and b using simultaneous equations.

$$\sum P(X=x) = 1 \Rightarrow 0.2+0.1+a+b+0.2=1$$

$$a+b=0.5 \quad ①$$

$$E(X)=2 \Rightarrow \sum xP(X=x) = 0\times0.2+1\times0.1+2\times a+3\times b+4\times0.2=2$$

$$\Rightarrow 0.1+2a+3b+0.8=2$$

$$2a+3b=1.1 \quad ②$$

① into ② $\Rightarrow 2(0.5-b)+3b=1.1$

$$\therefore \begin{cases} b = 0.1 \\ a = 0.4 \end{cases}$$

comments

Done well.

b) Hence calculate the standard deviation, correct to two decimal places. **(1)**

* Can use a calculator or use formula on ref. sheet.

Mark	Criteria
1	Obtains the correct value for the standard deviation.

Using formula:

$$\sigma^2 = E(X^2) - \mu^2$$

$$= \sum x^2 P(X=x) - \mu^2$$

$$= 0 + 1^2\times0.1 + 2^2\times0.4 + 3^2\times0.1 + 4^2\times0.2 - 2^2$$

$$\sigma^2 = 1.8 \Rightarrow \sigma \doteqdot 1.34 \quad ①$$

Question 13 (2 marks)

Show that the curve $y=3x^2-5\ln x$ is concave up for all values of $x>0$. **(2)**

$$\frac{dy}{dx}=6x-\frac{5}{x}$$

$$\frac{d^2y}{dx^2}=6+\frac{5}{x^2} \quad ①$$

Since $x^2>0$ for all $x>0$

$$\Rightarrow \frac{d^2y}{dx^2}=6+\frac{5}{x^2}>0 \text{ for all } x \quad ①$$

$\therefore$ concave up for all $x>0$.

Mark	Criteria
2	Correctly finds the second derivative and successfully uses this to conclude that the function is concave up for all $x>0$.
1	Finds the 2^{nd} derivative but fails to justify the concavity.

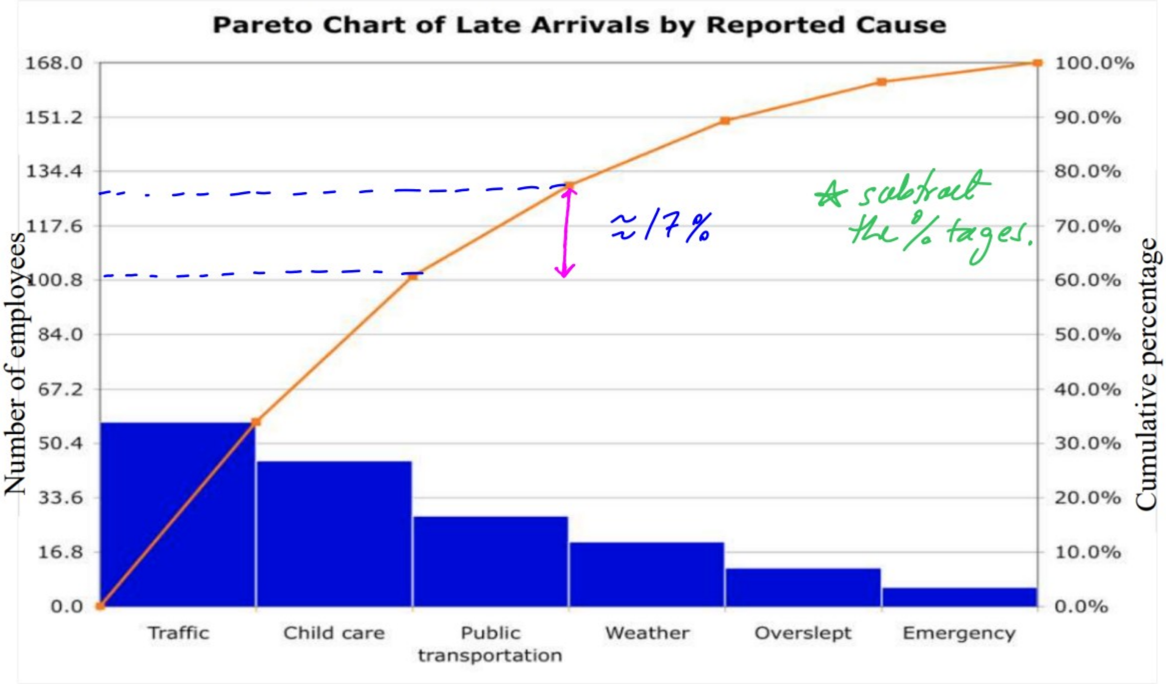
Comments

Some students incorrectly used the 1st derivative rather than the 2nd.

Question 14 (1 marks)

A company owner collected data related to the reasons given by employees for being late to work.

The Pareto chart shows the data collected.



≈17%

* subtract the %tages.

Estimate the percentage of employees that were late because of public transport. **(1)**

≈ 17% - 20%

Question 15 (2 marks)

Differentiate $e^{4x}\ln(x^2)$. **(2)**

$$\frac{d}{dx}\left(e^{4x}\times 2\ln x\right) = \frac{d}{dx}\left(2e^{4x}\ln x\right)$$

$$= 2\times4e^{4x}\ln x + 2e^{4x}\times\frac{1}{x} \quad ①$$

$$= 8e^{4x}\ln x + \frac{2e^{4x}}{x} \quad ①$$

OR $= 2e^{4x}\left(4\ln x + \frac{1}{x}\right)$

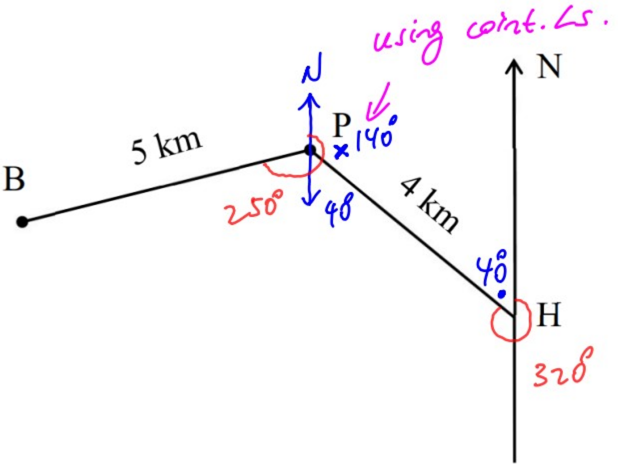
Mark	Criteria
2	Correctly uses the product rule and writes the derivative in the form $u'v+v'u$
1	Partially correct use of the product rule.

Comments

Done well.

Question 16 (5 marks)

Kate starts riding her bike from her house H. She rides 4 km on a bearing 320° to the park P then rides 5 km on a bearing 250° to the beach B.



i) Show that the angle HPB is 110°. **(1)**

$\angle HPB = 250° - 140° = 110°$

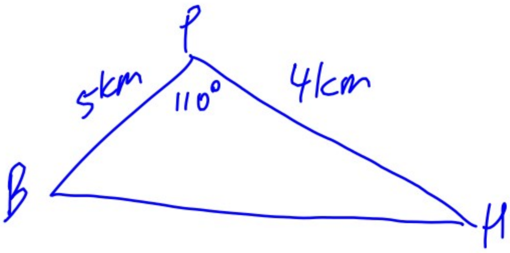
students must use the diagram to 'show.'

Mark	Criteria
1	**Shows** clearly why $< HPB$ is 110° by marking the alternate/co-interior angles subtended by the compass directions on the diagram.

Comments

A number of students lost marks by not clearly showing how they obtained the angle.

b) Find the distance BH correct to three decimal places. **(2)**



Using cosine Rule,

$$BH^2 = 5^2 + 4^2 - 2\times5\times4\cos110^\circ$$ ①

$$BH^2 = 41 - 40\cos110^\circ$$

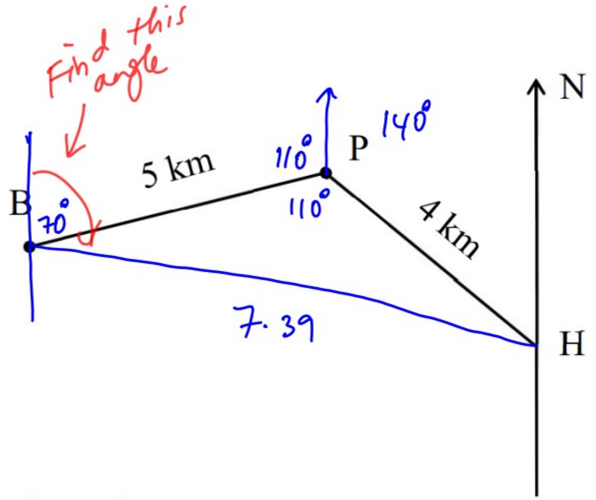
$$BH = \sqrt{41-40\cos110^\circ}$$

$$\doteqdot 7.39 \text{ km}$$ ①

Mark	Criteria
2	Successfully uses the cosine rule to find the correct distance of BH.
1	Uses the cosine rule but fails to find obtain the correct value of BH.

Comments
Some students had their calculators in radian mode instead of degrees.

c) Find the bearing of Kate's house H from the beach B. **(2)**



Mark	Criteria
2	Successfully uses the sine or cosine rule to find <HBP and then correctly uses this to find the bearing.
1	Shows elements of successful working towards the solutions.

Comments
Some students incorrectly used 90° to calculate their bearing.

$$\frac{\sin\angle HBP}{4} = \frac{\sin 110}{7.39} \Rightarrow \angle HBP = \sin^{-1}\left(\frac{4\sin110^\circ}{7.39}\right)$$

$$\doteqdot 31^\circ$$ ①

Bearing of H from B = $70^\circ + 31^\circ = 101^\circ$ T ①

Question 17 (3 marks)

If $0 \le \theta \le 2\pi$, solve $\sqrt{2}\cos\left(2\theta - \frac{\pi}{3}\right) - 1 = 0$.

Mark	Criteria
3	Finds all correct values for θ within the specified domain.
2	Find correct values of θ but misses certain solutions. **Or** Incorrectly simplifies the trig equation but finds all the correct solutions.
1	Simplifies the equation to $\cos\left(2\theta - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

Adjust Domain

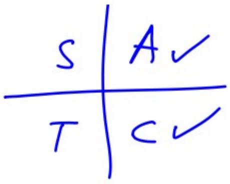
$0 \le \theta \le 2\pi$

$0 \le 2\theta \le 4\pi$

$-\frac{\pi}{3} \quad -\frac{\pi}{3} \quad -\frac{\pi}{3}$

$-\frac{\pi}{3} \le 2\theta - \frac{\pi}{3} \le \frac{11\pi}{3}$

$\cos\left(2\theta - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$ ①



$2\theta - \frac{\pi}{3} = -\frac{\pi}{4}, \frac{\pi}{4}, \underbrace{2\pi - \frac{\pi}{4}}_{=\frac{7\pi}{4}}, \underbrace{2\pi + \frac{\pi}{4}}_{=\frac{9\pi}{4}}$

$2\theta = \frac{\pi}{12}, \frac{7\pi}{12}, \underbrace{\frac{7\pi}{4} + \frac{\pi}{3}}_{=\frac{25\pi}{12}}, \underbrace{\frac{9\pi}{4} + \frac{\pi}{3}}_{=\frac{31\pi}{12}}$ ①

all $[\div 2]$

$\theta = \frac{\pi}{24}, \frac{7\pi}{24}, \frac{25\pi}{24}, \frac{31\pi}{24}$ ①

Comments

Many students did not modify the domain which led to students missing solutions.

Question 18 (2 marks)

Explain why the geometric series shown below does not have a limiting sum. **(2)**

$$3 + \frac{3}{\sqrt{3}-1} + \frac{3}{\left(\sqrt{3}-1\right)^2} + \ldots..$$

Mark	Criteria
2	States the value of the common ratio r and correctly justifies why the series does not have a limiting sum.
1	Only manages to correctly state the value of r.

$r = \frac{1}{\sqrt{3}-1} \doteqdot 1.37$ ①

Since $r > 1$, therefore there is no limiting sum ①

Note: There is only a limiting sum if $-1 < r < 1$.

Comments

Done well.

Question 19 (2 marks)

The future value of an annuity when \$1 is invested at the start of each period is shown in the table below.

Future value of \$1						
Number of periods	Interest rate per period					
	1%	2%	3%	4%	5%	6%
1	\$1.0100	\$1.0200	\$1.0300	\$1.0400	\$1.0500	\$1.0600
2	\$2.0301	\$2.0604	\$2.0909	\$2.1216	\$2.1525	\$2.1836
3	\$3.0604	\$3.1216	\$3.1836	\$3.2465	\$3.3101	\$3.3746
4	\$4.1010	\$4.2040	\$4.3091	\$4.4163	\$4.5256	\$4.6371
5	\$5.1520	\$5.3081	\$5.4684	\$5.6330	\$5.8019	\$5.9753
6	\$6.2135	\$6.4343	\$6.6625	\$6.8983	\$7.1420	\$7.3938

Jamie deposits \$800 into a savings account at the start of each month for 6 months. After the 6th deposit, Jamie stops making deposits but leaves the money in the savings account until exactly 12 months from the first deposit.
The interest rate on her savings account is 12% per annum, compounded monthly.
What is the balance of Jamie's savings account at the end of the 12 months? **(2)**

Future value of \$800 at the start of each month for 6 months = \$800 × 6.2135 (from table)

= \$4970.5 ①

After 6th deposit, Jamie stops deposits

i.e Value at end of 12 months (Amount invested for 6 months) $= \$ 4970.5 \times 1.01^6 \doteqdot 5276.29$ ①

Mark	Criteria
2	Finds the correct Future Value (FV) and then correctly uses the compound interest formula to find the final balance.
1	Finds the FV correctly but incorrectly finds the final balance. **Or** Fails to find the FV properly but correctly finds the final balance using the compound interest formula (carry error).

Comments

Done poorly. Many students failed to use the table properly.

Question 20 (6 marks)

Given a function:

$$f(x)=\begin{cases}\frac{3x}{4}(2-x) \text{ for } 0\le x\le 2\\ 0 \text{ otherwise}\end{cases}$$

i) Prove that $f(x)$ represents a probability density function. **(2)**

i.e show $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^2 \frac{3x}{4}(2-x)\,dx = \int_0^2 \frac{3}{4}(2x-x^2)\,dx$$

$$= \left[\frac{3}{4}\left(x^2-\frac{1}{3}x^3\right)\right]_0^2 \quad ①$$

$$= \frac{3}{4}\left(\left(4-\frac{8}{3}\right)-0\right)$$

$$= 1 \quad ①$$

$\therefore f(x)$ is a PDF.

Mark	Criteria
2	Shows via integration that $\int_{-\infty}^{\infty} f(x)\,dx = 1$ and hence is a PDF.
1	Correctly integrates the function but fails at correctly arriving at 1 upon substitution. **Or** Show elements of successful working towards the solution.

Comments

Most students obtained at least 1 mark.

ii) State the mode of the distribution. **(1)**

$f(x)=\frac{3x}{4}(2-x)$ is a conc. down parabola.

Hence mode = Axis of symmetry.

mode $= \frac{0+2}{2} = 1$ ①

☆ Average of x-ints.

Mark	Criteria
1	States the correct modal score.

Comments

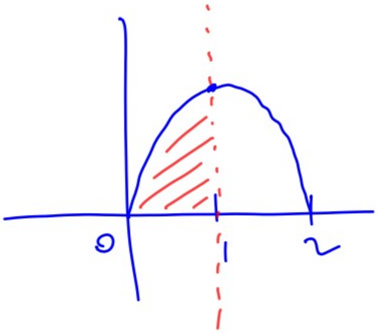
Some students incorrectly stated the function value $f(1)$ rather than just the score $x=1$.

iii) Explain why the median is equal to the mode in this distribution. **(1)**

Since the distribution is symmetric/symmetrical about $x = 1$, the mode must equal to the median.

Mark	Criteria
1	Correctly justifies that the distribution is symmetrical/symmetric about the mode and thus the mode must equal to the median.

See below



Comments

Students had to 'explain' rather than 'show' with a brief sentence. Stating that the function is 'even', 'parabolic' or a 'normal distribution' was not a valid response.

iv) Find $P(X > 1.5)$. **(2)**

PDF function

$$P(X>1.5) = \int_{1.5}^{2} f(x)\,dx = \left[\frac{3}{4}(2x - x^2)\right]_{1.5}^{2} \text{ from (i)}$$

$$= \frac{3}{4}\left((4-4) - (3\times1.5 - 1.5^2)\right)$$

$$= 0.15625 \text{ OR } \frac{5}{32}$$

Mark	Criteria
2	Successfully integrates the function to find the probability.
1	Writes the correct expression for the definite integral but fails at arriving at the correct answer. **Or** Finds the complement of this event.

Comments

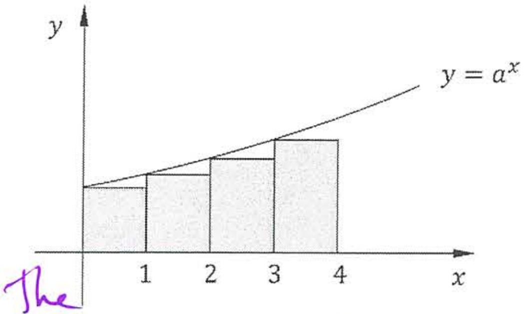
Some students found the the complement of this probability.

Question 21 (4 marks)

The diagram shows the graph of $y = a^x$, where $a > 1$. Also shown on the diagram are 4 inner rectangles of width 1 unit.

Also had to mention the G.P with r = a
a = 1

Couldn't use the show to get the formula for area.



NOT TO SCALE

i) Show that the area of the 4 rectangles is $\frac{a^4 - 1}{a - 1}\ u^2$. **(2)**

$A = 1 \times 1 + 1 \times a + 1 \times a^2 + 1 \times a^3$

$= 1 + a + a^2 + a^3$ G.P with a = 1

$= \frac{1(a^4 - 1)}{a - 1}$ r = 4

$= \frac{a^4 - 1}{a - 1}$

To get full marks you had to show how to get the area of each triangle

ii) By finding the area under the curve $y = a^x$ between $x = 0$ and $x = 4$, and using the result in part (i), prove that $\ln a < a - 1$. **(2)**

$A = \int_0^4 a^x \, dx$

$= \left[\frac{a^x}{\ln a} \right]_0^4$

$= \frac{a^4}{\ln a} - \frac{1}{\ln a}$

$\frac{a^4 - 1}{a - 1} < \frac{a^4 - 1}{\ln a}$

$\frac{a - 1}{a^4 - 1} > \frac{\ln a}{a^4 - 1}$

$a - 1 > \ln a$

Question 22 (4 marks)

A continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} 3^{-x}\ \ln 3 & \text{for } x \geq 0 \\ 0 & \text{for all other values of } x \end{cases}$$

i) Find the cumulative distribution function $F(x)$. **(2)**

$$\int_0^x 3^{-x} \ln 3 \, dx$$

$$= \left[\frac{-3^{-x}}{\cancel{\ln 3}} \cdot \cancel{\ln 3}\right]_0^x$$

$$= -3^{-x} + (3^0)$$

$$F(x) = -3^{-x} + 1$$

Many students couldn't do this question

ii) Find the exact value of the third quartile. **(2)**

$$F(x) = 0.75$$

$$-3^{-x} + 1 = \frac{3}{4}$$

$$3^{-x} = \frac{1}{4}$$

$$\ln 3^{-x} = \ln \frac{1}{4}$$

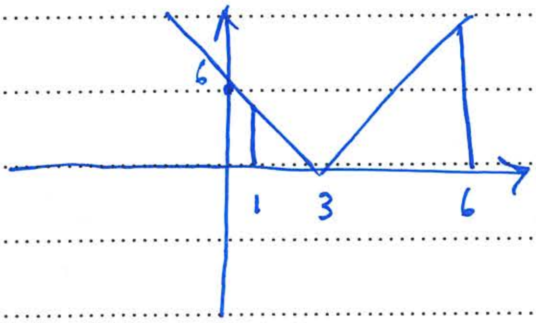
$$-x = \frac{\ln \frac{1}{4}}{\ln 3}$$

$$-x = \frac{\ln 4^{-1}}{\ln 3}$$

$$\boxed{x = \frac{\ln 4}{\ln 3}}$$

Question 23 (2 marks)

Evaluate $\int_1^6 |2x-6| \, dx$. **(2)**



$$\int_1^6 |2x-6| = \frac{1}{2}(2)(4) + \frac{1}{2} \times 3 \times 6$$

$$= 4 + 9$$

$$= 13$$

This question was done poorly. It is better to draw a diagram.

Question 24 (4 marks)

Sketch the graph of the curve $y = \dfrac{5x^2 e^{\frac{x}{2}}}{3}$ showing the coordinates of the turning points and their nature. **(4)**

$$y' = \frac{10x}{3} e^{\frac{x}{2}} + \frac{1}{2} e^{\frac{x}{2}} \times \frac{5x^2}{3}$$

$$= \frac{5x}{3} e^{\frac{x}{2}} \left(2 + \frac{1}{2}x\right)$$

This question was hard for many students They didn't realise it was exponential..

$$y' = 0$$

$\frac{5x}{3} = 0 \therefore x = 0 \longrightarrow y = 0 \quad (0,0)$

$e^{\frac{x}{2}} \neq 0 \qquad 2 + \frac{1}{2}x = 0$

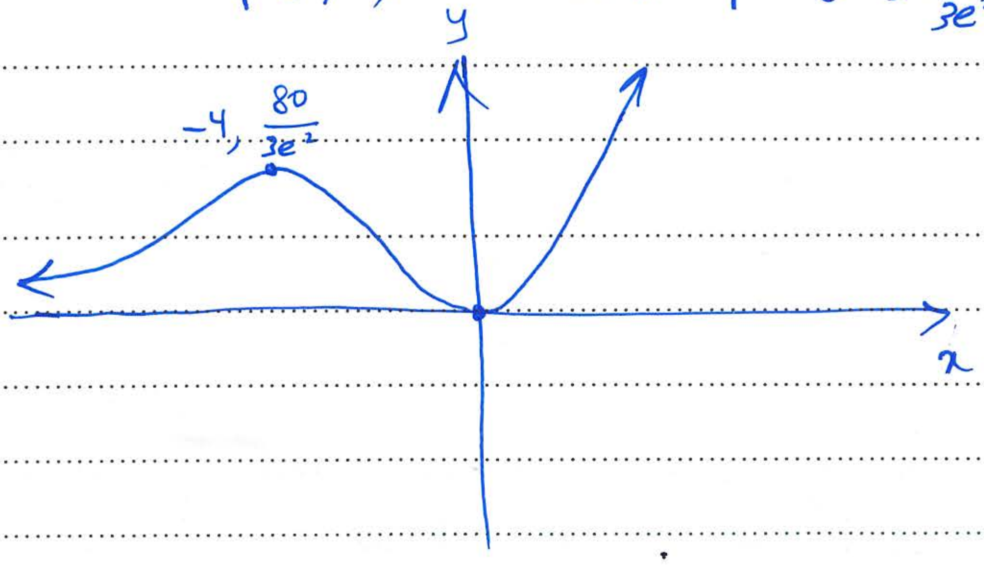
$2 = -\frac{1}{2}x \quad x = -4 \quad \left(-4, \frac{80}{3e^2}\right)$

x	-1	0	1
y'	-1.5	0	6.9

x	-5	-4	-1
y'	0.06	0	-1.5

min t.p (0,0)

Max t.p $\left(-4, \frac{80}{3e^2}\right)$



Question 25 (2 marks)

Differentiate $\ln\left(3\sin\frac{x}{2}\right)^3$. $= 3\ln\left(3\sin\frac{x}{2}\right)$ * many students forgot to do this. **(2)**

$\frac{d}{dx} 3\ln\left(3\sin\frac{x}{2}\right) = \frac{3 \times \cancel{3} \times \frac{1}{2}\cos\frac{x}{2}}{\cancel{3}\sin\frac{x}{2}}$ $\frac{f'(x)}{f(x)}$

$= \frac{3}{2}\cot\frac{x}{2}$

(This is the easiest method and most simplified answer)

Alternatively this answer was accepted.

$\frac{3\left(3\sin\frac{x}{2}\right)^2 \times \frac{3}{2}\cos\frac{x}{2}}{\left(3\sin\frac{x}{2}\right)^3}$

it also simplifies to

$= \frac{3}{2}\cot\frac{x}{2}$

Question 26 (2 marks)

If $\int_0^3 g(x)\,dx = 11$, find $\int_0^3\left(\frac{1}{2}g(x)+3x\right)dx$. **(2)**

$= \int_0^3 \frac{1}{2}g(x)dx + \int_0^3 3x\,dx$

$= \frac{1}{2}\int_0^3 g(x)\,dx + \left[\frac{3x^2}{2}\right]_0^3$

$= \frac{1}{2} \times 11 + \frac{3}{2}(9)$

$= \frac{11}{2} + \frac{27}{2}$ (1 mark awarded for a correct answer at this stage)

$= 19$

Question 27 (3 marks)

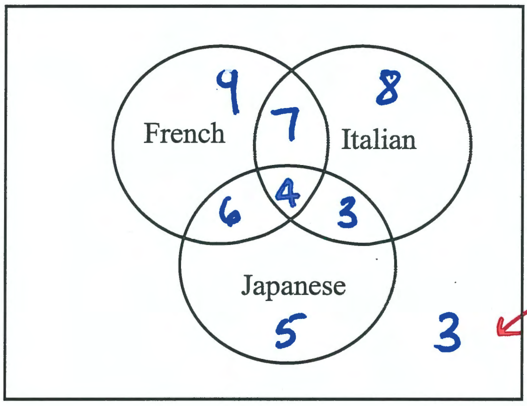
Last Sunday, 45 people were surveyed about three types of restaurants that they prefer to attend on their next birthday.

Of them, 26 chose French, 22 chose Italian, 18 Japanese.

It is also found that 11 chose French and Italian, 10 chose French and Japanese and 7 chose Italian and Japanese.

Only 4 people chose all three types of restaurants.

a) Represent this information in the Venn Diagram below. **(1)**



A large no. of students missed this and lost the mark. This did not affect the remaining questions.

i) A person from those who were surveyed last Sunday is to be selected at random. What is the probability that this person chose French or Italian but not Japanese? **(1)**

$$= \frac{9+7+8}{45}$$

$$= \frac{24}{45}$$

$$= \frac{8}{15}$$

ii) Two people are chosen at random, what is the probability of both chose French only? **(1)**

$$= \frac{9}{45} \times \frac{8}{44}$$

← many students made a mistake with the 2nd person.

$$= \frac{2}{55}$$

Question 28 (3 marks)

Prove that $\left(\cot\theta+\operatorname{cosec}\theta\right)^2=\dfrac{1+\cos\theta}{1-\cos\theta}$. **(3)**

$$L.H.S = \left(\frac{\cos Q}{\sin Q}+\frac{1}{\sin Q}\right)^2$$

$$=\left(\frac{\cos Q+1}{\sin Q}\right)^2$$

$$=\frac{(\cos Q+1)^2}{\sin^2 Q}$$

$$=\frac{(\cos Q+1)^2}{1-\cos^2 Q}$$

$$=\frac{(\cos Q+1)\cancel{(\cos Q+1)}}{(1-\cos Q)\cancel{(1+\cos Q)}}$$

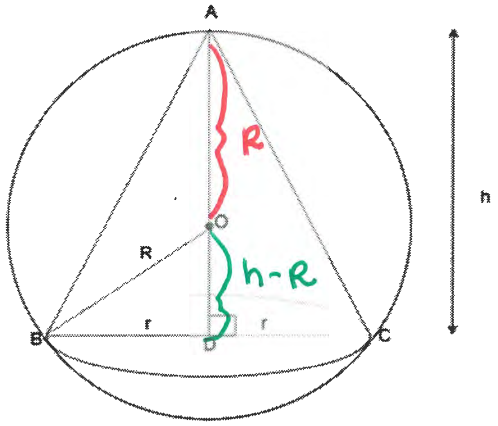
$$=\frac{1+\cos Q}{1-\cos Q}$$

$$=RHS$$

Even if students gained full marks the solution above should be considered. A lot of students overcomplicated the process.

Question 29 (6 marks)

Consider the following diagram with the cone inscribed in a sphere.



i) If r is the radius of the cone show that $r^2 = 2hR - h^2$. **(1)**

$r^2 + (h-R)^2 = R^2$

$r^2 + h^2 - 2hR + \cancel{R^2} = \cancel{R^2}$

$r^2 = 2hR - h^2$

Must be shown

ii) Show that the volume of the cone that can be inscribed in a sphere of radius R is given by $V = \frac{1}{3}\pi\left(2h^2R - h^3\right)$ where h is the height of the inscribed cone. **(1)**

$V = \frac{1}{3}\pi r^2 h$

From(i) $r^2 = 2hR - h^2$ → substitute

$V = \frac{1}{3}\pi(2hR - h^2)h$

$= \frac{1}{3}\pi(2h^2R - h^3)$

Must be shown

Question 29 continued on the next page

Question 29 continued

iii) Show that the volume of the largest cone is $\frac{8}{27}$ of the volume of the sphere. **(4)**

$V_S = \frac{4}{3}\pi R^3$

$V_C = \frac{1}{3}\pi\left(2h^2R - h^3\right)$

$= \frac{1}{3}\pi h^2(2R - h)$

$\frac{dV_c}{dh} = \frac{1}{3}\pi\left(4Rh - 3h^2\right)$ $\frac{d^2V_c}{dh^2} = \frac{1}{3}\pi(4R - 6h)$

let $\frac{dV_c}{dh} = 0$

$4Rh - 3h^2 = 0$

$4Rh = 3h^2$

$h^2 = \frac{4Rh}{3}$

$h^2 - \frac{4Rh}{3} = 0$

$h\left(h - \frac{4R}{3}\right) = 0$

$h = 0$ $h = \frac{4R}{3}$ ✓

(not a soln, $h > 0$)

when $h = \frac{4R}{3}$ $\frac{d^2V_c}{dh^2} = \frac{1}{3}\pi\left(4R - 6\left(\frac{4R}{3}\right)\right)$

$= \frac{1}{3}\pi(4R - 8R)$

$= -\frac{4\pi R}{3}$

Must be tested to determine its a maximum.

< 0 since $R > 0$: max at $\frac{4R}{3}$

<u>Question 29 continued</u>

iii) Show that the volume of the largest cone is $\frac{8}{27}$ of the volume of the sphere. **(4)**

continued.

For max volume Let $h = \frac{4R}{3}$

$$V_c = \frac{1}{3}\pi\left(2R\left(\frac{4R}{3}\right)^2 - \left(\frac{4R}{3}\right)^3\right)$$

$$= \frac{1}{3}\pi\left(\frac{32R^3}{9} - \frac{64R^3}{27}\right)$$

$$= \frac{1}{3}\pi\left(\frac{96R^3 - 64R^3}{27}\right)$$

$$= \frac{1}{3}\pi \times \frac{1}{27} \times 32R^3$$

$$= \frac{1}{3}\pi\left(\frac{32R^3}{27}\right)$$

$$= \frac{32\pi R^3}{81}$$ ✓ $$V_s = \frac{4}{3}\pi R^3$$

$$V_c \div V_s = \frac{\overset{8}{\cancel{32}}\cancel{\pi}\cancel{R^3}}{\cancel{81}\,27} \times \frac{\cancel{3}}{\cancel{4}\cancel{\pi}\cancel{R^3}}$$

$$= \frac{8}{27}$$ MUST BE SHOWN

∴ showing volume of largest cone is $\frac{8}{27}$ volume of sphere.

Question 30 (2 marks) (The table on page 27 can be used for this question)

Madelyn is a long jumper training for the national age championships. To be able to compete at the national age championships her longest jump must be able to be beaten by less than 0.17% of the general population.
The distance jumped by the general population is normally distributed with a mean of 3.48 metres and standard deviation of 1.08 metres.
What is the minimum distance that Madelyn needs to jump, correct to the nearest centimetre?

$\bar{x} = 3.48$

$\sigma = 1.08$

$Z = \frac{D - 3.48}{1.08}$ $\quad D > Z \times 1.08 + 3.48$ *

$P(Z < z) > 1 - 0.17\%$

$> 1 - 0.0017$

> 0.9983

$\therefore z = 2.93$

- Most students found correct 'z' score. ①

$\therefore d > \bar{x} + 2.93\sigma$

$> 3.48 + 2.93 \times 1.08$

> 6.644

- Most students were able to find * distance correctly using 'z' score.

$\therefore$ Minimum distance

$= 6.64$ m (nearest cm) ①

- 1 mark for correct 'z' score
- 1 mark for correct distance
- 1 mark for using incorrect 'z' score to find distance correctly.

Table : The standard normal distribution

The table below provides some values of the probabilities for the standard normal distribution.

$$\text{i.e.}\quad \Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \phi(t)dt$$

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56360	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

Method 1

– For both methods

Question 31 (4 marks)

Determine the area of the region enclosed by $y^2 = 2x + 6$ and $y = x - 1$.

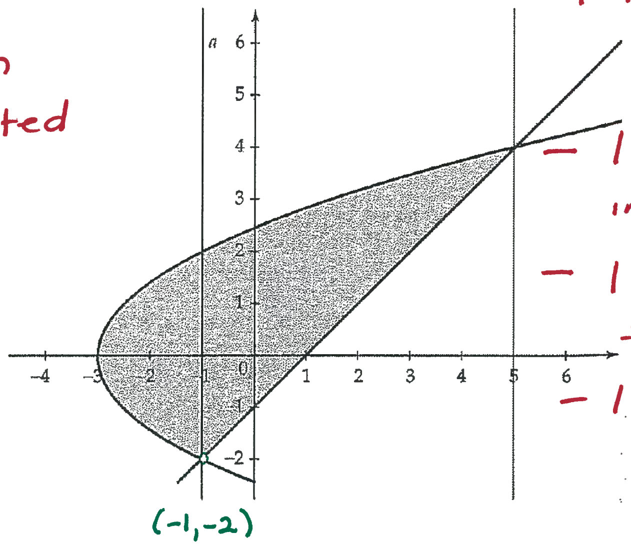
This question was completed poorly.

– 1 mark for correct integral

– 1 mark for correct integration.

– 1 mark for correct substitution

– 1 mark for correct answer.



$$\text{Area} = 2\int_{-3}^{-1} \sqrt{2x+6}\,dx + \int_{-1}^{5} \sqrt{2x+6} - (x-1)\,dx \quad ①$$

$$= 2\int_{-3}^{-1} (2x+6)^{1/2}\,dx + \int_{-1}^{5} (2x+6)^{1/2} - x + 1\,dx$$

$$= 2.\frac{\cancel{2}}{3}.\frac{1}{\cancel{2}}(2x+6)^{3/2}\Big]_{-3}^{-1} + \frac{\cancel{2}}{3}.\frac{1}{\cancel{2}}(2x+6)^{3/2} - \frac{x^2}{2} + x\Big]_{-1}^{5} \quad ①$$

$$= \frac{2}{3}(2x+6)^{3/2}\Big]_{-3}^{-1} + \frac{1}{3}(2x+6)^{3/2} - \frac{x^2}{2} + x\Big]_{-1}^{5}$$

$$= \left[\frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)\right] + \left[\left(\frac{1}{3}(16)^{3/2} - \frac{25}{2} + 5\right) - \left(\frac{1}{3}(4)^{3/2} - \frac{1}{2} - 1\right)\right] \quad ①$$

$$= \frac{2}{3}.8 + \left[\left(\frac{64}{3} - \frac{25}{2} + 5\right) - \left(\frac{8}{3} - \frac{3}{2}\right)\right]$$

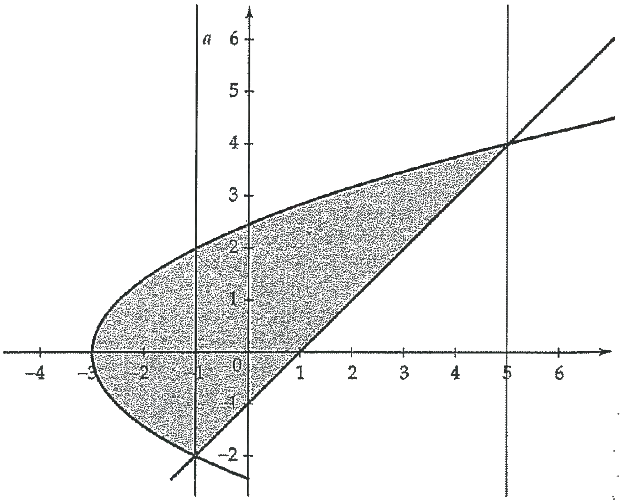
$$= \frac{16}{3} + \frac{83}{6} - \frac{7}{6}$$

$$= 18 \text{ units}^2. \quad ①$$

Method 2

Question 31 (4 marks)

Determine the area of the region enclosed by $y^2 = 2x + 6$ and $y = x - 1$.



This was the preferred method by most students.

Alternative method around the y-axis

$$\text{Area} = \int_{-2}^{4} (y+1) - \left(\frac{y^2}{2} - 3\right) dy \quad ①$$

$$= \int_{-2}^{4} y - \frac{y^2}{2} + 4 \; dy$$

$$= \left.\frac{y^2}{2} - \frac{y^3}{6} + 4y\right]_{-2}^{4} \quad ①$$

$$= \left[\frac{4^2}{2} - \frac{4^3}{6} + 4(4)\right] - \left[\frac{(-2)^2}{2} - \frac{(-2)^3}{6} + 4(-2)\right]$$

$$= \left[8 - \frac{32}{3} + 16\right] - \left[2 + \frac{8}{6} - 8\right] \quad ①$$

$$= \frac{40}{3} + \frac{14}{3}$$

$$= \frac{54}{3} = 18 \text{ units}^2 \quad ①$$

Question 32 (4 marks)

Madison is learning to drive. Her first lesson is 10 minutes long. Her second lesson is 15 minutes long. Each subsequent lesson is 5 minutes longer than the previous lesson.

i) How long will Madison's fifteenth lesson be? **(1)**

$10 + 15 + 20 + 25 \ldots$

$d = 15 - 10 = 20 - 15$ $\qquad a = 10$

$d = 5$ $\qquad T_n = a + (n-1)d$

$T_{15} = 10 + (15-1) \times 5$

$T_{15} = 10 + 14 \times 5$

$T_{15} = 80$ minutes long.

(Nearly all students got this correct)

①

- To obtain the 1 mark T= 80mins was only accepted.

ii) How many hours of lessons will Madison have completed after her fifteenth lesson? **(1)**

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{15} = \frac{15}{2}\left[2(10) + (15-1)5\right]$

$= \frac{15}{2}\left[20 + 70\right]$

$= 675$ minutes $\div 60$

$= 11 \cdot 25$ hours

$= 11$ hours and 15 minutes

①

(Most students got this correct)

-To obtain the 1 mark T=11 hours 15 mins was the only answer accepted.

iii) During which lesson will Madison have completed a total of 40 hours of driving lessons? **(2)**

$S_n = 40$ hours $\qquad 40 \times 60 = 2400$ mins.

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$2400 = \frac{n}{2}\left[20 + (n-1)5\right]$

$4800 = n\left[20 + 5n - 5\right]$

$4800 = 15n + 5n^2$

$\therefore n^2 + 3n - 960 = 0$

$n = \frac{-3 \pm \sqrt{9 - 4(1)(-960)}}{2}$

$n = \frac{-3 \pm \sqrt{3849}}{2}$

$\therefore n = \frac{-3 + \sqrt{3849}}{2}$

$n = \frac{-3 + 62 \cdot 04}{2}$

$n = 29 \cdot 5$

$n = 30$

①

∴ During Madison's 30th lesson she would of completed 40 hours of driving

- 1 mark for quadratic equation

- 1 mark for n=30th

①

Method 1: Using Change of base

Question 33 (2 marks)

Solve the equation $\log_2(2-2x) = \log_{\sqrt{2}}(1-x)$. **(2)**

$\log_2(2-2x) = \log_{\sqrt{2}}(1-x)$

$\frac{\ln(2-2x)}{\ln 2} = \frac{\ln(1-x)}{\ln\sqrt{2}}$

$\frac{\ln(2-2x)}{\ln 2} = \frac{\ln(1-x)}{\frac{1}{2}\ln 2}$

$\frac{\ln(2-2x)}{\ln 2} = \frac{2\ln(1-x)}{\ln 2}$

$\ln(2-2x) = 2\ln(1-x)$

$2-2x = (1-x)^2$ — 1 mark for quadratic

$2-2x = 1-2x+x^2$

$1 = x^2$

$x = \pm 1$ — 1 mark for correct solution

Testing each value of x

only solution $x=-1$.

- Most students used this method.

- Overall question was completed well, although a lot of students weren't able to get $x=-1$ as the only solution

Method 2 :

Question 33 (2 marks)

Solve the equation $\log_2(2-2x) = \log_{\sqrt{2}}(1-x)$. (2)

$\log_a y = x$

$y = a^x$

let $y = \log_2 (2-2x)$ .. ①

$y = \log_{\sqrt{2}} (1-x)$ .. ②

From ①

Now, $2-2x = 2^y$

$2(1-x) = 2^y$

$1-x = \frac{1}{2} . 2^y$

$1-x = 2^{-1} . 2^y$

$1-x = 2^{y-1}$ .. ①

From ②

$1-x = 2^{\frac{1}{2} y}$ .. ②

①

let ① = ②

$2^{y-1} = 2^{\frac{1}{2} y}$

$y-1 = \frac{1}{2} y$

$y - \frac{1}{2} y = 1$

$\frac{1}{2} y = 1$

$y = 2$. sub into ①.

$1-x = 2^{y-1}$

$1-x = 2^{2-1}$

$1-x = 2$

$-x = 2-1$

$-x = 1$

$\therefore x = -1$. ①

– 1 mark for showing $2^{y-1} = 2^{\frac{1}{2} y}$

– 1 mark for $x = -1$ as the only solution.

Question 34 (2 marks)

Differentiate $y = \left(\sin^3 2x^\circ\right)$. (2)

$2\pi = 360^\circ$

$\pi = 180^\circ$

$\therefore 1^\circ = \pi/180$

$y = \sin^3 \frac{2\pi}{180} x$

$y = \left(\sin \frac{\pi}{90} x\right)^3$

$y' = 3\left(\sin \pi/90\, x\right)^2 \times \cos \frac{\pi}{90} x \times \frac{\pi}{90}$

$y' = \frac{\pi}{30} . \left(\sin \frac{\pi}{90} x\right)^2 . \cos \frac{\pi}{90} x$. ①

– 1 mark for changing degrees in terms of π ①

– 1 mark for correct solution.

– This was completed poorly as most students didn't substitute $1^\circ = \pi/180$.

– No marks were given for differentiating y without being in terms of π.

Question 35(i)	Marks
· Provides correct solution.	1 mark

Solution

$$
\begin{aligned}
A_1 &= 50000 - M\\
A_2 &= A_1 - M\\
&= (50000 - M) - M\\
&= 50000 - 2M\\
A_3 &= A_2 - M\\
&= (50000 - 2M) - M\\
&= 50000 - 3M\\
&\vdots\\
A_6 &= 50000 - 6M \checkmark
\end{aligned}
$$

Comments

- To prove formulae in financial maths, derive expressions for A_1, A_2 and A_3 first.
- It was necessary to establish the pattern of repeated subtraction for full marks.

Question 35(ii)	Marks
· Provides correct solution.	2 marks
· Finds an expression for A_7 in terms of M only.	1 mark

Solution

$$
\begin{aligned}
A_7 &= A_6 \times 1.005 - M\\
&= (50000 - 6M) \times 1.005 - M \checkmark\\
A_8 &= A_7 \times 1.005 - M\\
&= [(50000 - 6M) \times 1.005 - M] \times 1.005 - M\\
&= (50000 - 6M) \times 1.005^2 - M(1.005) - M\\
&= (50000 - 6M) \times 1.005^2 - M(1.005 + 1) \checkmark
\end{aligned}
$$

Comments

- Students who skipped the crucial line $A_8 = A_7 \times 1.005 - M$ lost one mark.
- It is not enough to mentally expand the brackets, you have to show every step.

Question 35(iii)	Marks
· Provides correct solution.	2 marks
· Finds expression for A_{120} in terms of M only.	1 mark

Solution

Continue the pattern, then use the sum of a geometric series.

$$\begin{aligned}
A_{120} &= (50000-6M)(1.005)^{114} - (1.005^{113} + 1.005^{112} + \cdots + 1.005 + 1)M \ \checkmark \\
&= (50000-6M)(1.005)^{114} - \frac{1.005^{114}-1}{1.005-1}M \\
&= (50000-6M)(1.005)^{114} - 200(1.005^{114}-1)M \ \checkmark
\end{aligned}$$

Comments

- · This part was quite well done.
- · A common error was to write one of the powers as 120, when in fact both are 114.
- · In cases where this mistake was made, one mark was lost in (iii).
- · If (iv) was then answered correctly, full marks were awarded in (iv) (carried error).

Question 35(iv)	Marks
· Provides correct solution.	2 marks
· Sets $A_{120} = 0$ and attempts to solve for M.	1 mark

Solution

We want the value of M such that $A_{120} = 0$.

$$\begin{aligned}
A_{120} &= 0 \\
(50000-6M)(1.005)^{114} - 200(1.005^{114}-1)M &= 0 \\
50000(1.005)^{114} - 6M(1.005)^{114} - 200(1.005^{114}-1)M &= 0 \ \checkmark \\
50000(1.005)^{114} &= 6M(1.005)^{114} + 200(1.005^{114}-1)M \\
50000(1.005)^{114} &= M\left[6(1.005)^{114} + 200(1.005^{114}-1)\right] \\
M &= \frac{50000(1.005)^{114}}{6(1.005)^{114} + 200(1.005^{114}-1)} \\
M &\approx 539.18 \ \checkmark \ \text{(nearest cent)}
\end{aligned}$$

Comments

- · A common error was to set $A_{120} = 50000$ instead of $A_{120} = 0$.
- · Algebraic errors also occurred when attempting to solve for the equation for M.

Question 36(i)	**Marks**
· Provides correct solution.	2 marks
· Attempts to differentiate using the product rule.	1 mark

Solution

The derivative of a constant is 0, so we have

$$\begin{aligned}\frac{dx}{dt} &= \frac{d}{dt}\left(4 + 3te^{-2t}\right)\\ &= \frac{d}{dt}(3te^{-2t}).\end{aligned}$$

Now use the product rule on $3te^{-2t}$:

$$\begin{aligned} u &= 3t & \qquad v &= e^{-2t}\\ u' &= 3 & \qquad v' &= -2e^{-2t} \checkmark\end{aligned}$$

Therefore

$$\begin{aligned}\frac{dx}{dt} &= vu' + uv'\\ &= e^{-2t} \times 3 + 3t \times \left(-2e^{-2t}\right)\\ &= 3e^{-2t} - 6te^{-2t}\\ &= 3e^{-2t}\left(1 - 2t\right) \checkmark\end{aligned}$$

Comments

- For “show” questions, it is not enough to mentally differentiate using the product rule.
- Students must show they are not just working backwards from the answer.
- One mark was awarded when students showed $vu' + uv'$ by bracketing the expression.
- For example, $\frac{dx}{dt} = (e^{-2t})(3) + (3t)(-2e^{-2t})$.
- It was necessary to state $u = 3t, u' = 3, v = e^{-2t}, v' = -2e^{-2t}$ for full marks.

Question 36(ii)	**Marks**
· Provides correct solution.	1 mark

Solution

$$\begin{aligned}\text{When } t = \tfrac{1}{2},\ v &= 3e^{-2(\frac{1}{2})}\left(1 - 2(\tfrac{1}{2})\right)\\ &= 3e^{-1}\left(1 - 1\right)\\ &= 0. \checkmark\end{aligned}$$

Hence the particle is at rest when $t = \frac{1}{2}$.

Comments

- Well done.

Question 36(iii)	**Marks**
· Provides correct solution.	2 marks
· Calculates distance travelled in first half of journey, namely $\frac{3}{2e}$ metres.	1 mark

Solution

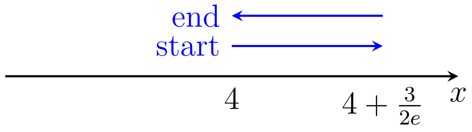
The initial position of the particle is $x = 4$.
The only time the particle is at rest is when $t = \frac{1}{2}$.
At this time, the displacement is

$$\begin{aligned} x &= 4 + 3(\tfrac{1}{2})e^{-2(\frac{1}{2})} \\ &= 4 + \tfrac{3}{2e} \end{aligned}$$

In the limit as $t \to \infty$, we get

$$\begin{aligned} x &= 4 + \tfrac{3t}{e^{2t}} \\ &\to 4 + 0 \\ &= 4 \end{aligned}$$

Therefore, the particle starts at $x = 4$, travels right to $x = 4 + \frac{3}{2e}$, then travels left back to $x = 4$ without ever reaching it. This can be visualised on the number line below:



The distance travelled when moving right is $(4 + \frac{3}{2e}) - 4 = \frac{3}{2e}$. ✓
The distance travelled when moving left is also $(4 + \frac{3}{2e}) - 4 = \frac{3}{2e}$.
It follows that the greatest total distance is

$$\tfrac{3}{2e} + \tfrac{3}{2e} = \tfrac{3}{e}. \checkmark$$

Comments

- This part was very challenging.
- Students needed to find the initial position, maximum displacement and final position.
- Finding these three points was necessary for one mark.
- Alternatively, one mark awarded for finding that $\frac{3}{2e}$ was the distance in the first half.
- Other approaches involved integration or sketching displacement against time.
- The solution above is simpler than these other approaches.
- Students are encouraged to visualise journeys on the number line like above.

Question 37	**Marks**
· Provides correct solution.	3 marks
· Successively finds A_{16} and A_{17} or an expression for A_{35} with working.	2 marks
· Finds A_{15}.	1 mark

Solution

$$\begin{aligned}
A_1 &= 7200(1.07)\\
A_2 &= (A_1 + 7200)(1.07)\\
&= (7200(1.07) + 7200)(1.07)\\
&= 7200(1.07)^2 + 7200(1.07)\\
&\vdots\\
A_{15} &= 7200(1.07)^{15} + 7200(1.07)^{14} + \cdots + 7200(1.07)^2 + 7200(1.07)\\
&= \frac{7200(1.07)(1.07^{15} - 1)}{1.07 - 1}\\
&= 193593.9855\ldots \checkmark
\end{aligned}$$

From this point, the contributions are \$10000 at the start of each year:

$$\begin{aligned}
A_{16} &= (A_{15} + 10000)(1.07)\\
&= A_{15}(1.07) + 10000(1.07)\\
A_{17} &= (A_{16} + 10000)(1.07)\\
&= A_{15}(1.07)^2 + 10000(1.07)^2 + 10000(1.07) \checkmark\\
&\vdots\\
A_{35} &= A_{15}(1.07)^{20} + 10000(1.07)^{20} + 10000(1.07)^{19} + \cdots + 10000(1.07)\\
&= A_{15}(1.07)^{20} + \frac{10000(1.07)(1.07^{20} - 1)}{1.07 - 1}\\
&\approx \$1\,187\,799.41 \checkmark \quad \text{(nearest cent).}
\end{aligned}$$

Comments

- This question was challenging.
- Some students defined A_n as the value of the fund at the start of year n.
- In that case, the required value was A_{36} (start of year 36 = end of year 35).
- Errors were made with geometric series, using wrong value of n or forgetting 1.07.
- Some students used a nice alternative method.
- They kept track of each individual payment, then added.
- For example, the first investment of 7200 grows to $7200(1.07)^{35}$.